POISSON NOISE REDUCTION WITH NON-LOCAL PCA

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PROBLEM FORMULATION

Setting: Poisson noise

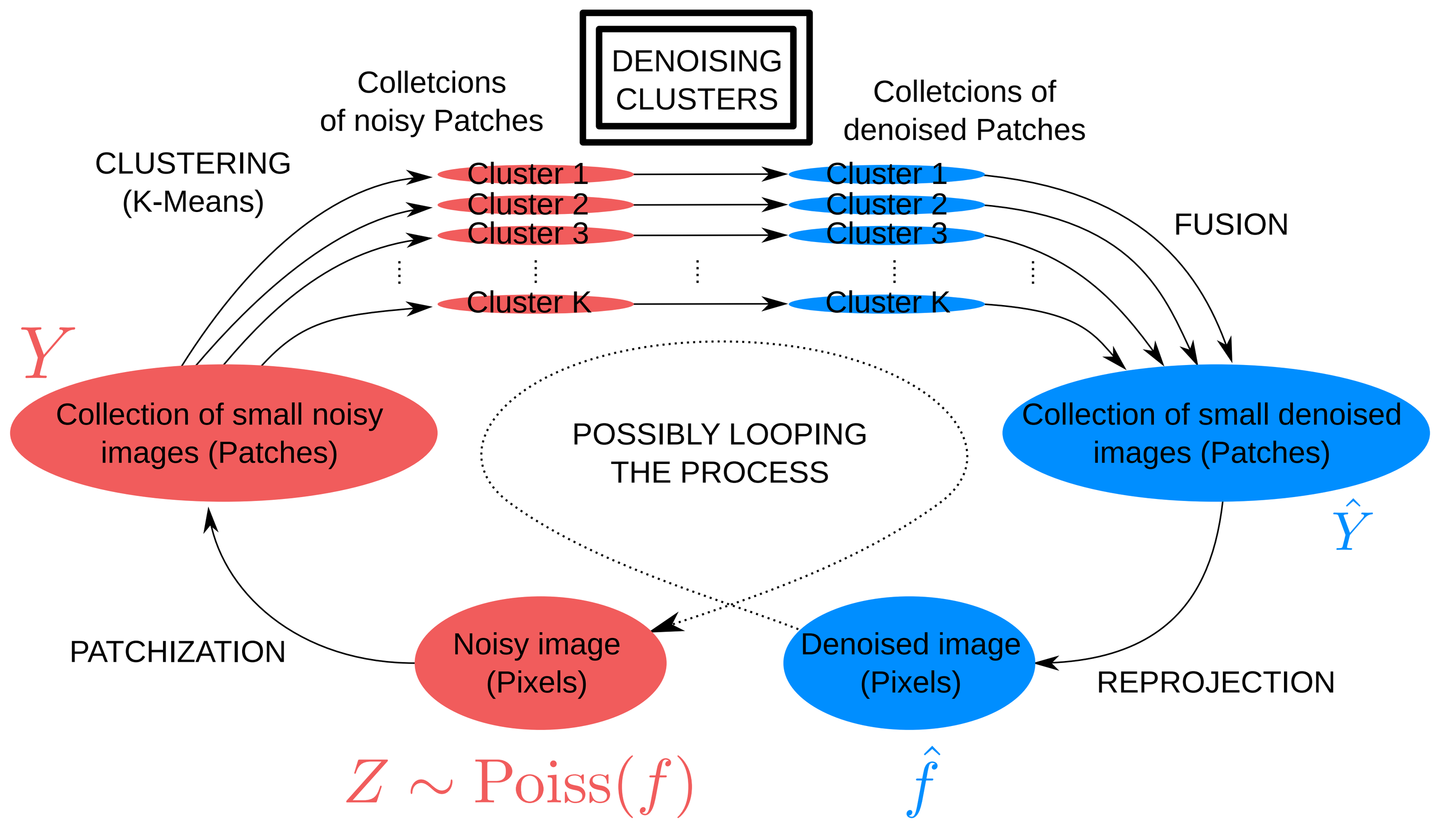
Observations : M independent observed pixel values Z_i

Quantities to be recovered : intensities $f_i > 0$

Noise generation : the Z_i are Poisson variables with mean $f_i \geq 0$

$$\mathbb{P}(Z_i | f_i) = \frac{f_i^{Z_i} e^{-f_i}}{Z_i!} .$$

General scheme:



DENOISING CLUSTERS WITH POISSON-PCA

Y : $M \times N$ **matrix** of all the (vectorized) $\sqrt{N} \times \sqrt{N}$ overlapping patches extracted from the noisy image. Then, one aims to approximate Y by :

$$\forall (i,j) \in [\![1, M]\!] \times [\![1, N]\!] , \quad Y_{i,j} \approx \exp([UV]_{i,j}) \quad ,$$

- U : $M \times \ell$ matrix of coefficients.
- V : $\ell \times N$ matrix representing the dictionary components/axis

Under the low rank assumption: ℓ is small with respect to M: $\ell \ll M$.

The framework introduced in [4] leads to minimizing:

$$L(U,V) = \sum_{i=1}^{M} \sum_{j=1}^{N} \exp(UV)_{i,j} - Y_{i,j}(UV)_{i,j} \quad ,$$

with respect to U and V. Defining the corresponding minimizers

$$(U^*, V^*) = \underset{(U,V) \in \mathbb{R}^{M \times \ell} \times \mathbb{R}^{\ell \times N}}{\arg\min} L(U,V) ,$$

the original data is then denoised by considering $\hat{Y} = \exp(U^* V^*)$.

BENEFITS OF THE POISSON APPROACH

With respect to other methods the Poisson-PCA has several advantages:

- No positivity constraints in the optimization problem
- Direct approach, no Anscombe / stabilization transform as in [1, 3]
- More general than wavelet-based approaches

NEWTON'S METHOD FOR MINIMIZING L

Second order gradient desecent [5]: Hessian matrices are needed, with respect to variable U and V: $H_U = \nabla_U^2 L(U,V)$ and $H_V = \nabla_V^2 L(U,V)$.

$$[H_U]_{(a,b),(c,d)} = \begin{cases} \sum_{j=1}^{N} \exp(UV)_{a,j} V_{b,j}^2, & \text{if } (a,b) = (c,d), \\ 0 & \text{otherwise.} \end{cases}$$

$$[H_V]_{(a,b),(c,d)} = \begin{cases} \sum_{i=1}^{M} U_{i,a}^2 \exp(UV)_{i,b}, & \text{if } (a,b) = (c,d), \\ 0 & \text{otherwise.} \end{cases}$$

Updating the coefficients rows: $U_{t+1,i,:}$ is the ith row of U_{t+1}:

$$U_{t+1,i,:} = U_{t,i,:} - (\exp(U_t V_t)_{i,:} - Y_{i,:}) V_t^\top (V_t D_i V_t^\top)^{-1} , \qquad (1)$$

where $D_i = \mathrm{diag}\left(\exp(U_t V_t)_{i,1}, \ldots, \exp(U_t V_t)_{i,N}\right)$ is a diagonal matrix of size $N \times N$.

Updating the dictionary columns: $V_{t,:,j}$ is the jth column of V_t

$$V_{t+1,:,j} = V_{t,:,j} - (U_{t+1}^\top E_j U_{t+1})^{-1} U_{t+1}^\top (\exp(U_{t+1} V_t)_{:,j} - Y_{:,j}) , \qquad (2)$$

where $E_j = \mathrm{diag}\left(\exp(U_{t+1} V_t)_{1,j}, \ldots, \exp(U_{t+1} V_t)_{M,j}\right)$ is a diagonal matrix of size $M \times M$.

Rem: a conditioner $\varepsilon_{\mathrm{cond}}$ might be needed to invert the Hessian

Peak	Direct-2P	Ansc-2P	Direct-1P	Ansc-1P	Alg. in [2]
0.1	**20.31**	18.73	19.18	18.97	19.48
0.2	**22.49**	20.74	22.04	21.55	20.93
0.5	**25.58**	23.98	25.34	24.31	23.69
1	26.79	26.17	**26.88**	26.46	25.07
2	27.90	28.25	28.10	**28.34**	26.67
3	28.57	28.94	28.73	**29.12**	27.62
4	29.21	29.31	29.27	**29.61**	28.19

Table: PSNR for Saturn (ave. over ten realizations): NLPCA one/two pass (Direct-1P/Direct-2P), the Gaussian NL-PCA (Anscombe transform) with one/two pass (Ansc-1P/Ansc-2P) and haarTIApprox [2].

ALGORITHM

Input: noisy image Z ; **Output:** estimated image $\hat{f}$

Parameters: Patch size $\sqrt{N} \times \sqrt{N}$, number of clusters K, number of components ℓ, maximal number of iterations N_{iter}

Clusterization: create K clusters of patches using K-Means

for cluster k **do**

  Initialize $U_0 = \mathrm{randn}(M_k, \ell)$ and $V_0 = \mathrm{randn}(\ell, N)$

  while $t \leq N_{\mathrm{iter}}$ and test $> \varepsilon_{\mathrm{stop}}$ **do**

    for $i \leq M_k$ **do**

      Update the ith row of U using (1)

    end for

    for $j \leq \ell$ **do**

      Update the jth column of V using (2)

    end for

    $t := t + 1$

  end while

  $\hat{Y}^k = \exp(U_t V_t)$

end for

Reprojection: average the various pixel estimates due to overlaps

Second iteration: use the denoised image to improve clustering

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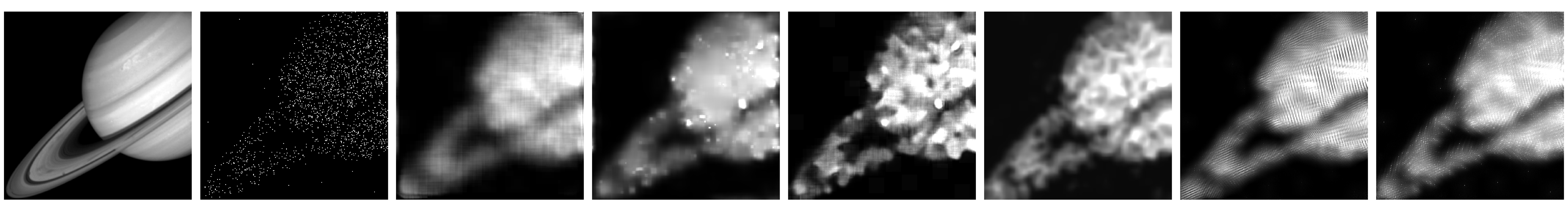
Online code : http://josephsalmon.eu/

VISUAL RESULTS



Original data Peak = 0.1 | Noisy data, PSNR=4.77 | SAFIR [1], PSNR=19.94 | HaarTIApprox [2], PSNR=19.23 | PLNM [6], PSNR=16.34 | BM3D [3], PSNR=18.76 | Gaussian NL-PCA, PSNR=17.84 | Poisson NL-PCA, PSNR=20.06

- $N \; = \; 20^2$: patch size
- $\ell \; = \; 4$: rank of approximation
- $K \; = \; 14$: number of clusters
- $\varepsilon_{\mathrm{stop}} = 10^{-1}$: stopping criterion
- $N_{\mathrm{iter}} = 20$: maximum iterations
- $\varepsilon_{\mathrm{cond}} = 10^{-3}$: Hessian conditionner