From Patches to Pixels in Non-Local Methods: Weighted-Average Reprojection (Wav)

Joseph Salmon*, Yann Strozecki
salmon@math.jussieu.fr*, strozecki@logique.jussieu.fr

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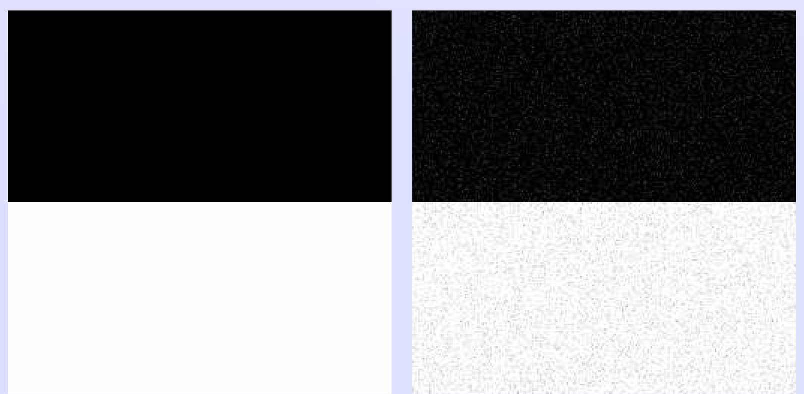
Noisy image (AWGN)



$$I(\mathbf{x}) = I^\star(\mathbf{x}) + \varepsilon(\mathbf{x})$$

$\mathbf{x} \in \Omega$: pixel in the image Ω, ε: centered Gaussian noise with known variance σ^2
Problem: denoise I

Non-Local Means (NLM)

Patches: square indexed by upper left corner, width W

$$\mathsf{P}^I_{\mathbf{x}} = (I(\mathbf{x}+\tau), \tau \in \{0,\dots,W-1\}^2)$$

NLM Denoiser:

$$\widehat{I}(\mathbf{x}) = \frac{\sum_{\mathbf{x}'} \theta^I(\mathbf{x},\mathbf{x}') I(\mathbf{x}')}{\sum_{\mathbf{x}''} \theta^I(\mathbf{x},\mathbf{x}'')}$$

$$\text{where} \quad \theta^I(\mathbf{x},\mathbf{x}') = K(\|\mathsf{P}^I_{\mathbf{x}} - \mathsf{P}^I_{\mathbf{x}'}\|/h)$$

$\mathbf{x}', \mathbf{x}'' \in \Omega_R(\mathbf{x})$ (searching window)
K: kernel function, $h > 0$ bandwidth
$\|\cdot\|$: Euclidean norm
Patch point of view:

$$\widehat{\mathsf{P}}^I_{\mathbf{x}} = \sum_{\mathbf{x}' \in \Omega_R(\mathbf{x})} \frac{\theta^I(\mathbf{x},\mathbf{x}') \cdot \mathsf{P}^I_{\mathbf{x}'}}{\sum_{\mathbf{x}'' \in \Omega_R(\mathbf{x})} \theta^I(\mathbf{x},\mathbf{x}'')}$$

Central-Reprojection: $(W = 2W_1 + 1)$

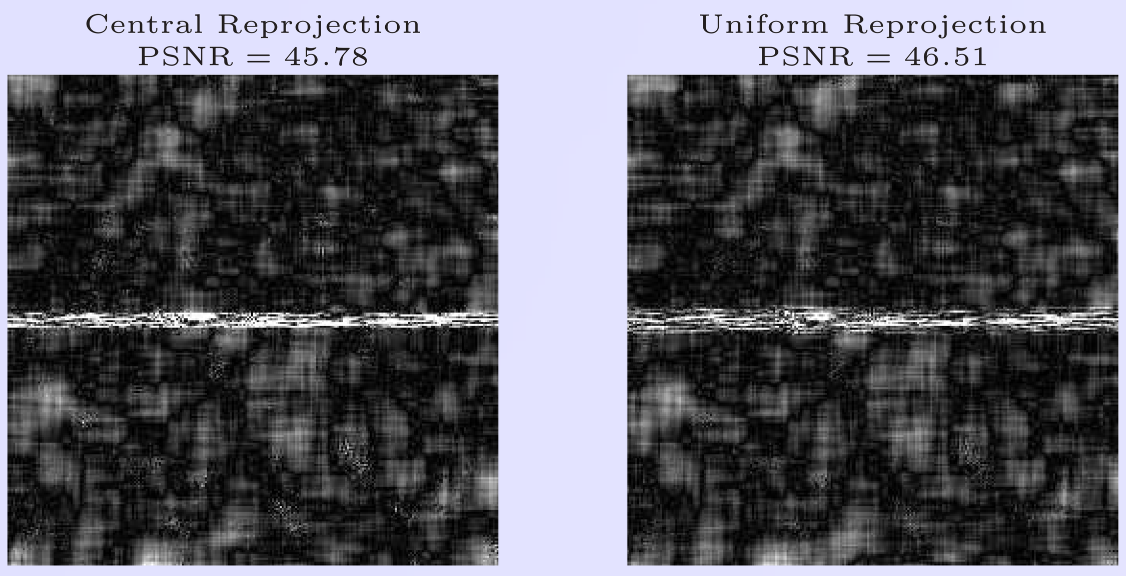
$$\widehat{I}_{\mathrm{Cent}}(\mathbf{x}) = \widehat{\mathsf{P}}^I_{\mathbf{x}-\delta_W}(\delta_W), \delta_W = (W_1, W_1)$$

Uniform-Reprojection:

$$\widehat{I}_{\mathrm{Uae}}(\mathbf{x}) = \frac{1}{W^2} \sum_{\delta \in \{0,\dots,W-1\}^2} \widehat{\mathsf{P}}^I_{\mathbf{x}-\delta}(\delta)$$

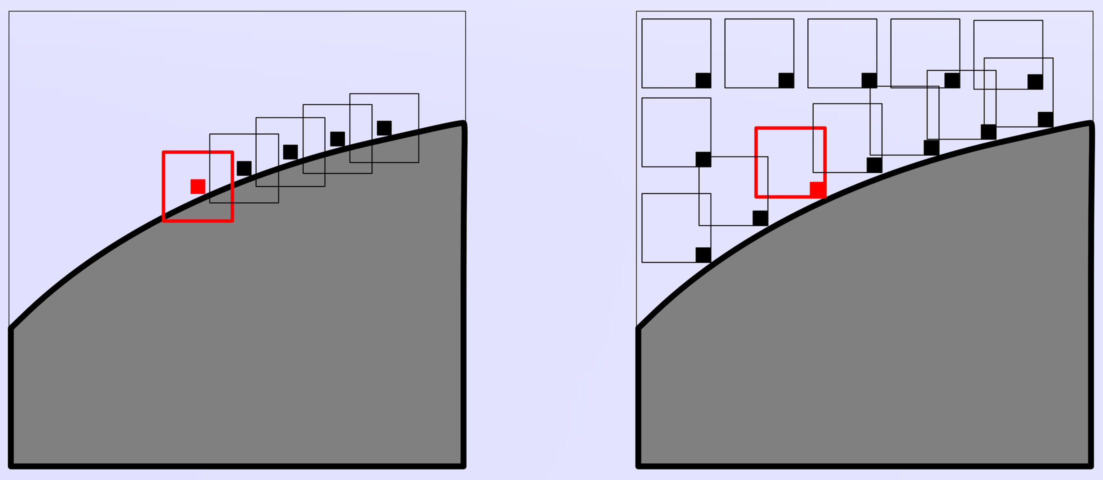
Halo artifacts along edges

Absolute difference $|\widehat{I} - I|$:



Number of similar patches $\Longleftrightarrow$ Variance

Sliding reprojections vs. halo



Why halo?

- linear number of similar patches near edges (**weak** similarity, **high** variance)
- quadratic number far from edges (**strong** similarity, **low** variance)

Solutions: weight the slided position as a function of the variance.

Variance-based reprojections

Slided estimate variance: $\mathrm{Var}\Big(\widehat{\mathsf{P}}^I_{\mathbf{x}-\delta}(\delta)\Big) \approx$

$$\sigma^2 \sum_{\mathbf{x}' \in \Omega_R(\mathbf{x})} \Big(\theta^I(\mathbf{x},\mathbf{x}')\Big)^2 \Big/ \Big(\sum_{\mathbf{x}' \in \Omega_R(\mathbf{x})} \theta^I(\mathbf{x},\mathbf{x}')\Big)^2$$

Min-Reprojection:

$$\widehat{I}_{\mathrm{Min}}(\mathbf{x}) = \widehat{\mathsf{P}}^I_{\mathbf{x}-\hat{\delta}}(\hat{\delta}),$$

$$\text{with} \quad \hat{\delta} = \mathop{\arg\min}_{\delta \in \{0,\dots,W-1\}^2} \mathrm{Var}\Big(\widehat{\mathsf{P}}^I_{\mathbf{x}-\delta}(\delta)\Big)$$

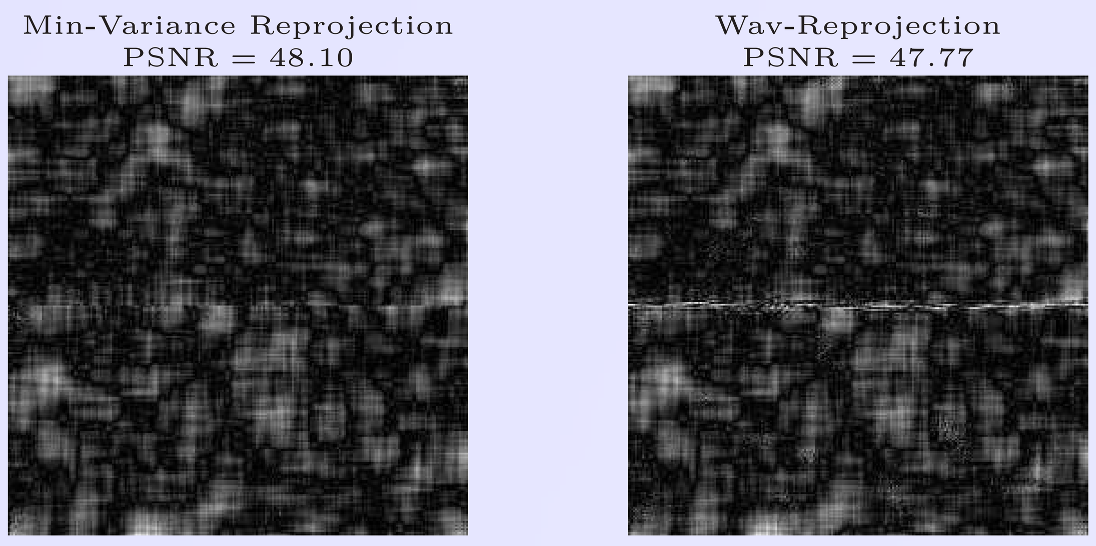
Wav-Reprojection: same minimization but for convex combination of estimates, closed formula:

$$\widehat{I}_{\mathrm{Wav}}(\mathbf{x}) = \sum_{\delta \in \{0,\dots,W-1\}^2} \beta_\delta \widehat{\mathsf{P}}^I_{\mathbf{x}-\delta}(\delta),$$

$$\text{with } \beta_\delta = \frac{\Big[\mathrm{Var}\Big(\widehat{\mathsf{P}}^I_{\mathbf{x}-\delta}(\delta)\Big)\Big]^{-1}}{\sum_{\delta \in \{0,\dots,W-1\}^2} \Big[\mathrm{Var}\Big(\widehat{\mathsf{P}}^I_{\mathbf{x}-\delta}(\delta)\Big)\Big]^{-1}}$$

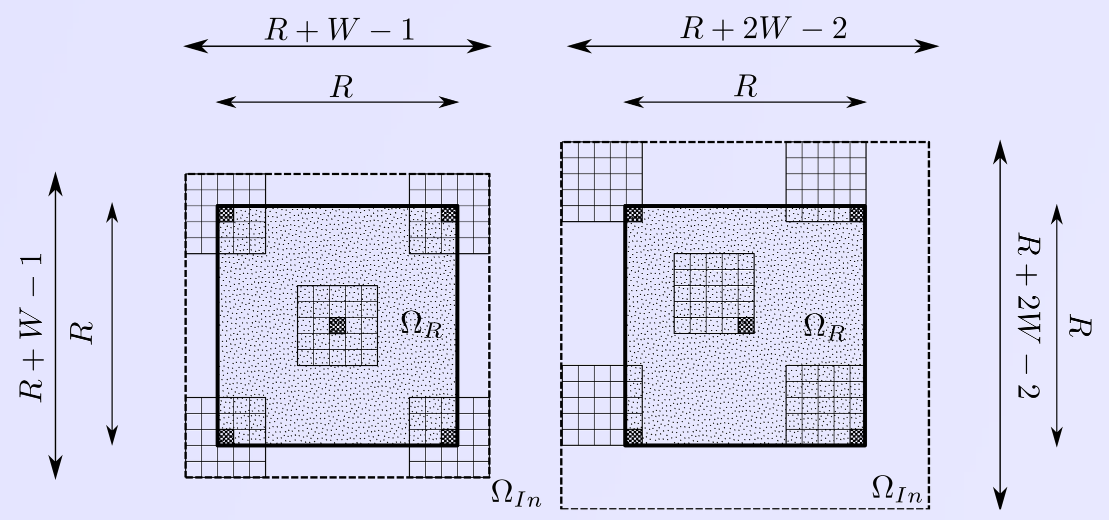
Flat kernel: $K(t) = \mathbb{1}_{[-1,1]}(t)$
Variance=1/number of similar patches

Halo reduction by Wav-reprojection



Implementation

Flat kernel: efficient implementation, only need to keep the number of "selected patches"
Searching zone: can be reduced thanks to sliding (speeds up the NLM)



Visual results

NLM variants: $R = 9$, $W = 9$, $\sigma = 20$
Gaussian kernel: $h = \sigma$
Flat kernel: $h^2 = 2\sigma^2 q^{W^2}_{0.99}$ (χ^2 quantile)

Online Matlab (mex) code:
www.math.jussieu.fr/~salmon/

References

A. Buades and B. Coll and J-M. Morel **A review of image denoising algorithms, with a new one** (2005)
A. Foi **Anisotropic nonparametric image processing: theory, algorithms and applications** (2005)

Conclusion

Wav-reprojection: PSNR increase between 0.1-0.5 dB, numerically efficient
Take-away message: think patch-wise and combine the slided NLM estimates, it **reduces halo artifacts**